Clipping and other geometric algorithms

MIT EECS 6.837
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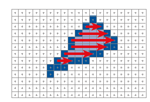
Final projects

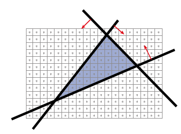
- Rest of semester
  - Weekly meetings with TAs
  - Office hours on appointment
- This week, with TAs
  - Refine timeline
  - Define high-level architecture
- Project should be a whole, but subparts should be identified with regular merging of code

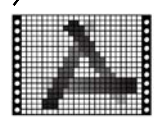
Review of last time?

Last time

- Polygon scan conversion
  - Smart
    - Take advantage of coherence
    - Good for big triangles
  - back to brute force
    - Incremental edge equation
    - Good for small triangles
    - Simpler clipping
- Visibility
  - Painer: complex ordering
  - Z buffer: simple, memory cost
    - Hyperbolic z interpolation

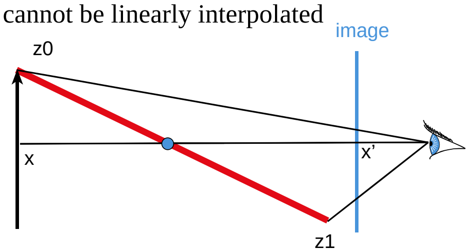






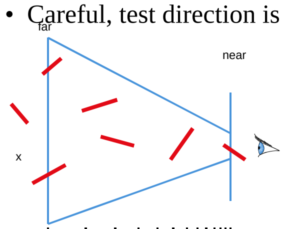
Z interpolation

- X'=x/z
- Hyperbolic variation
- Z cannot be linearly interpolated



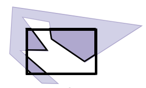
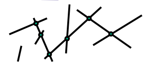
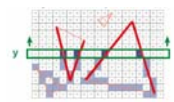
Integer z-buffer

- Use 1/z to have more precision in the foreground
- Set a near and far plane
  - 1/z values linearly encoded between 1/near and 1/far
- Careful, test direction is reversed



Plan

- Review of rendering pipeline
- 2D polygon clipping
- Segment intersection
- Scanline rendering overview

The Graphics Pipeline

Modeling Transformations
Illumination (Shading)
Viewing Transformation (Perspective / Orthographic)
Clipping
Projection (to Screen Space)
Scan Conversion (Rasterization)
Visibility / Display

Input:
Geometric model:
Description of all object, surface, and light source geometry and transformations
Lighting model:
Computational description of object and light properties, interaction (reflection)
Synthetic Viewpoint (or *Camera*):
Eye position and viewing frustum
Raster Viewport:
Pixel grid onto which image plane is mapped

Output:
Colors/Intensities suitable for framebuffer display
(For example, 24-bit RGB value at each pixel)

Modeling Transformations

Modeling Transformations
Illumination (Shading)
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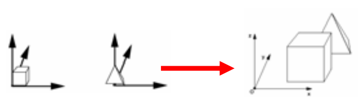
Visibility / Display



Object space World space

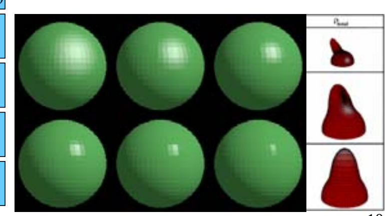
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Illumination (Shading) (Lighting)

Modeling Transformations
Illumination (Shading)
Viewing Transformation (Perspective / Orthographic)
Clipping
Projection (to Screen Space)
Scan Conversion (Rasterization)
Visibility / Display

- Vertices lit (shaded) according to material properties, surface properties (normal) and light
- Local lighting model (Diffuse, Ambient, Phong, etc.)

$$L(\omega_r) = k_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)\frac{\Phi_s}{4\pi\, d^2}$$



Viewing Transformation

Modeling Transformations
Illumination (Shading)
Viewing Transformation (Perspective / Orthographic)
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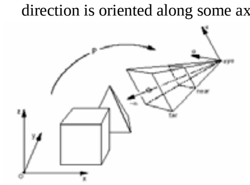
Visibility / Display

- Viewing position is transformed to origin & direction is oriented along some axis (usually z)



Eye space

World space

Yet another 4x4 matrix

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Clipping

Modeling Transformations
Illumination (Shading)
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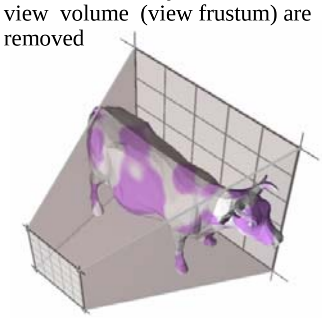
Visibility / Display

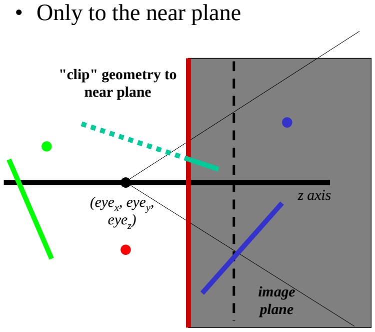
- Portions of the object outside the view volume (view frustum) are removed



Clipping – modern hardware

- Modeling Transformations
- Illumination (Shading)
- Viewing Transformation (Perspective / Orthographic)
- Clipping
- Projection (to Screen Space)
- Scan Conversion (Rasterization)
- Visibility / Display

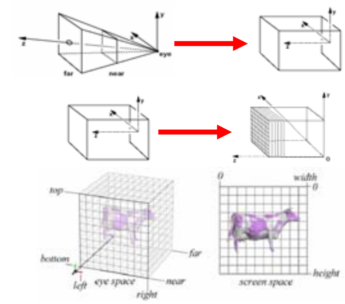
- Only to the near plane



Projection

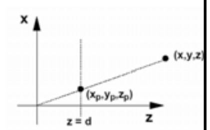
- Modeling Transformations
- Illumination (Shading)
- Viewing Transformation (Perspective / Orthographic)
- Clipping
- Projection (to Screen Space)
- Scan Conversion (Rasterization)
- Visibility / Display

- Projective transform



Perspective Projection

- 2 conceptual steps:
  - 4x4 matrix
  - Homogenize
    - In fact not always needed
    - Mordern graphics hardware performs most operations in 2D homogeneous coordinates

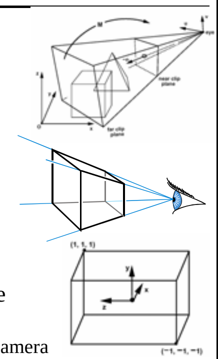


homogenize

$$\begin{pmatrix} x * d / z \\ y * d / z \\ d/z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \\ z / d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

When to clip?

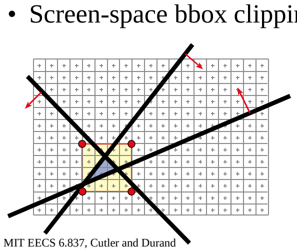
- Before perspective transform in 3D space
  - Use the equation of 6 planes
  - Natural, not too degenerate
- In homogeneous coordinates after perspective transform (Clip space)
  - Before perspective divide (4D space, weird *w* values)
  - Canonical,independent of camera
  - The simplest to implement in fact
- In the transformed 3D screen space after perspective division
  - Problem: objects in the plane of the camera



Scan Conversion (Rasterization)

- Modeling Transformations
- Illumination (Shading)
- Viewing Transformation (Perspective / Orthographic)
- Clipping
- Projection (to Screen Space)
- Scan Conversion (Rasterization)
- Visibility / Display

- Incremental edge equations
- Interpolate values as we go (color, depth, etc.)
- Screen-space bbox clipping



Visibility / Display

- Modeling Transformations
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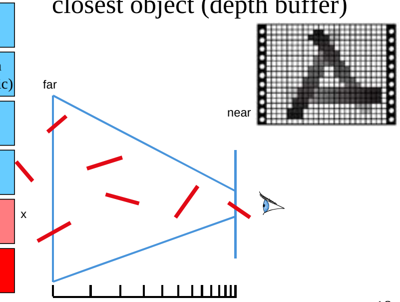
- Visibility / Display

- Each pixel remembers the closest object (depth buffer)



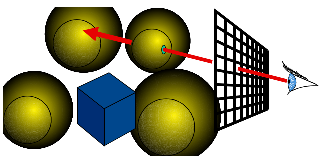
Rendering Pipeline vs. ray casting

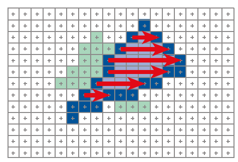
Ray Casting
```
For each pixel
   For each object
```
Send pixels to the scene

Discretize first

Rendering Pipeline
```
For each triangle
   For each pixel
```
Project scene to the pixels

Discretize last

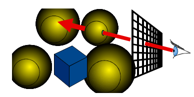


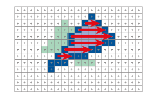


Rendering Pipeline vs. ray casting

Ray Casting
```
For each pixel
   For each object
```
- Depth complexity: no calculation for hidden part
- Whole scene must be in memory
- Very atomic computation
- More general, more flexible
  - Primitive, lighting effects, adaptive antialiasing

Rendering Pipeline
```
For each triangle
   For each pixel
```
- Coherence: geometric transforms for vertices only
- Arithmetic intensity: the amount of computation increases in the depth of the pipeline
  - Good bandwidth/computation ratio
- Harder to get global illumination (shadows, interreflection, etc.)

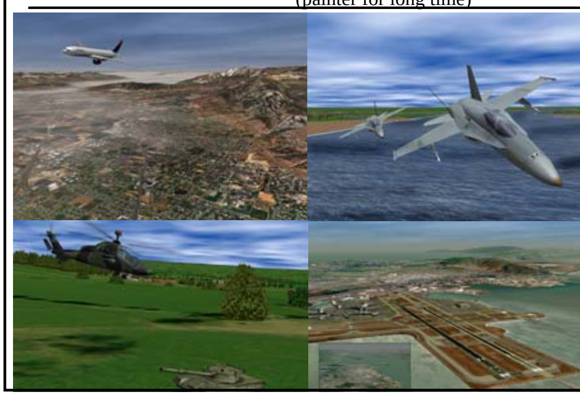




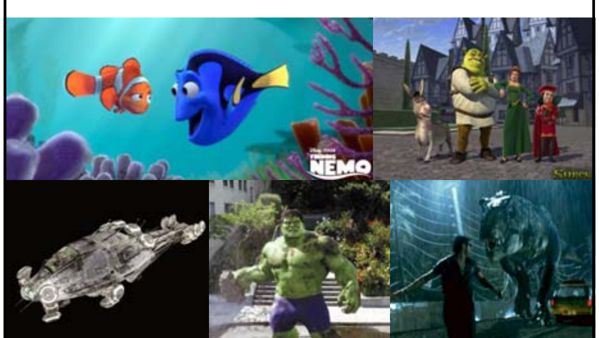
Games : pipeline

Flight simulation : pipeline

(painter for long time)

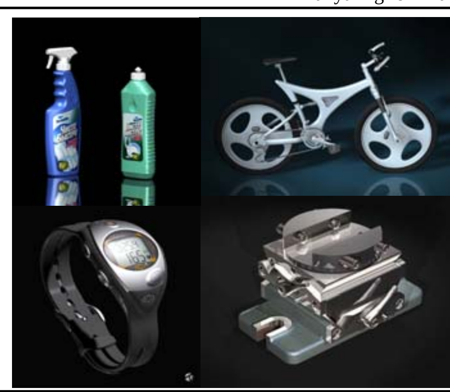


Movies : Both pipeline and ray tracing



CAD-CAM & design

pipeline during design, anything for final image



Architecture

ray-tracing, pipeline, but do complex lighting simulation (cf. later lectures)

Virtual reality : pipeline

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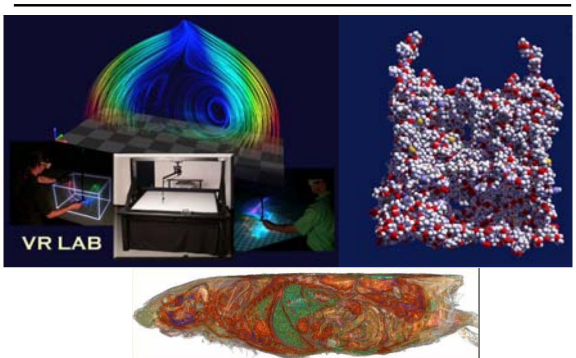
Visualization:

mostly pipeline, ray-tracing for high-quality eye candy, interactive ray-tracing is starting



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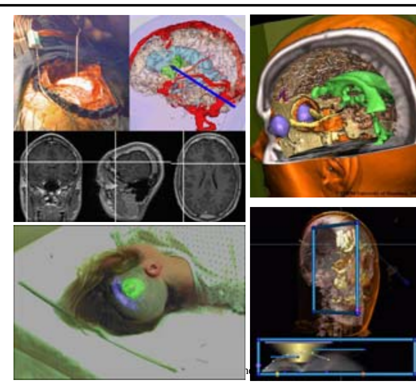
Medical imaging: cf. visualization



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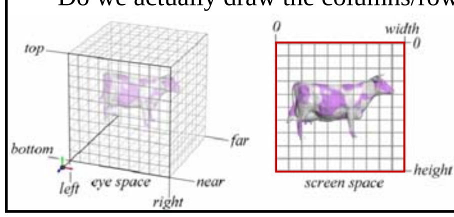
Questions?

The infamous half pixel

- I refuse to teach it, but it’s an annoying issue you should know about
- Do a line drawing of a rectangle from [top, right] to [bottom,left]
- Do we actually draw the columns/rows of pixels?



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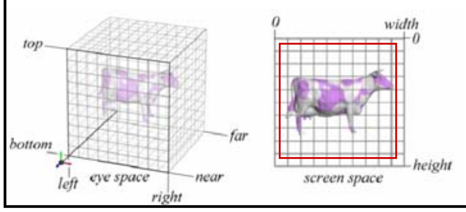
The infamous half pixel

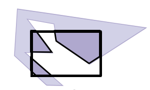
- Displace by half a pixel so that top, right, bottom, left are in the middle of pixels
- Just change the viewport transform



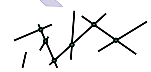
Plan

- Review of rendering pipeline

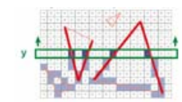
- 2D polygon clipping



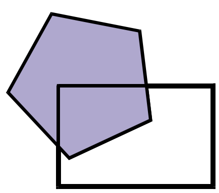
- Segment intersection



- Scanline rendering overview



Polygon clipping



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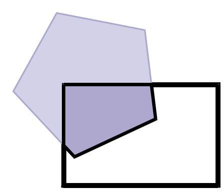
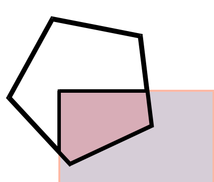


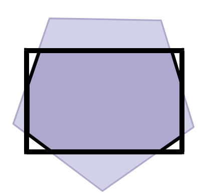
Polygon clipping

- Clipping is symmetric



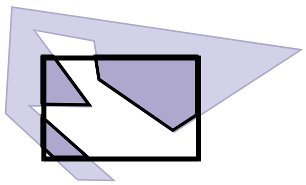
Polygon clipping is complex

- Even when the polygons are convex



Polygon clipping is nasty

- When the polygons are concave



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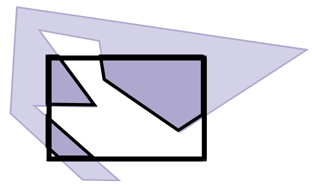
Naïve polygon clipping?

- N*m intersections
- Then must link all segment
- Not efficient and not even easy



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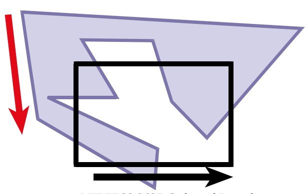
Weiler-Atherton Clipping

- Strategy: “Walk" polygon/window boundary
- Polygons are oriented (CCW)



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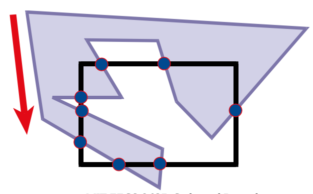
Weiler-Atherton Clipping

- Compute intersection points



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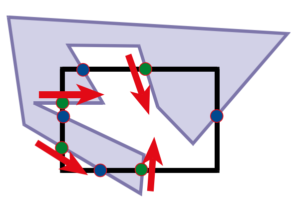
Weiler-Atherton Clipping

- Compute intersection points
- Mark points where polygons enters clipping window (green here)



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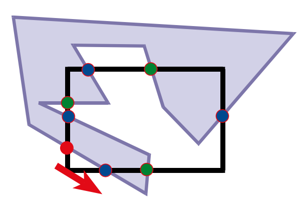
Clipping

While there is still an unprocessed entering intersection
Walk" polygon/window boundary



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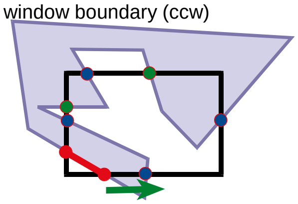
Walking rules

- Out-to-in pair:
  - Record clipped point
  - Follow polygon boundary (ccw)
- In-to-out pair:
  - Record clipped point
  - Follow window boundary (ccw)



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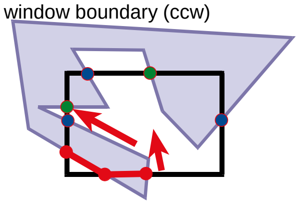

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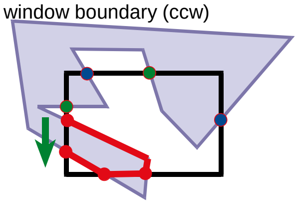

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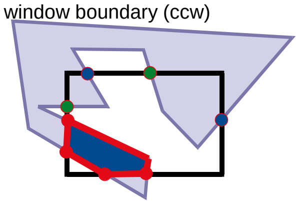

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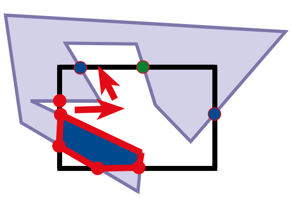
Walk" polygon/window boundary



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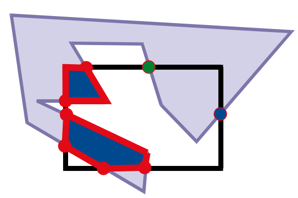

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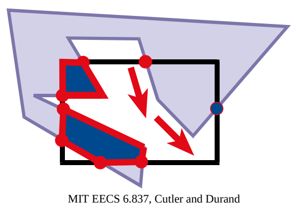

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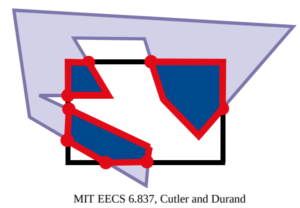
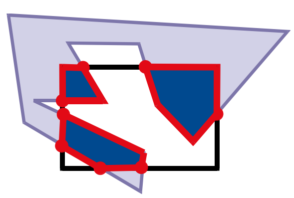


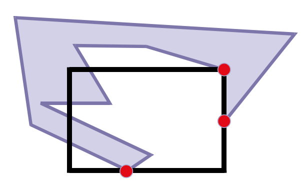
Weiler-Atherton Clipping

- Importance of good adjacency data structure (here simply list of oriented edges)



Robustness, precision, degeneracies

- What if a vertex is on the boundary?
- What happens if it is “almost” on the boundary?
  - Problem with floating point precision
- Welcome to the real world of geometry!

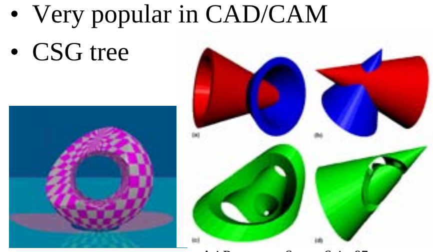


Clipping

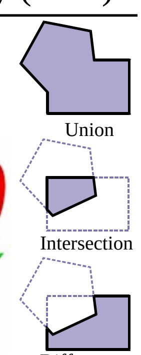
- Many other clipping algorithms:
- Parametric, general windows, region-region, One-Plane-at-a-Time Clipping, etc.

Constructive Solid Geometry (CSG)

- Sort of generalized clipping
- Boolean operations
- Very popular in CAD/CAM
- CSG tree



Ari Rappoport, Steven Spitz 97



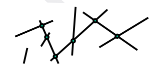
Questions?

Plan

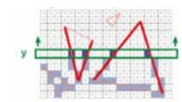
- Review of rendering pipeline

- 2D polygon clipping

- Segment intersection

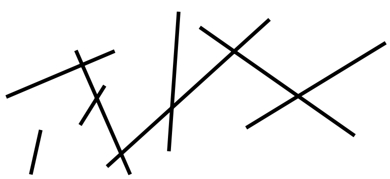


- Scanline rendering overview



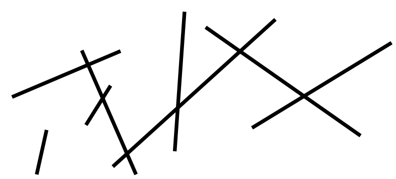
Line segment intersection

- N segments in the plane
- Find all intersections



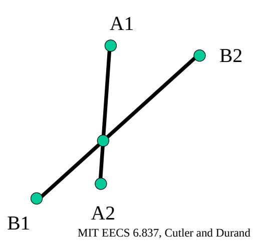
Maximum complexity?

- N^2
- (always N^2 if we take full lines)



Intersection between 2 segments

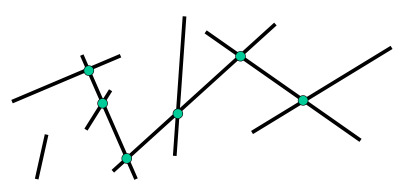
- Compute line equation for the 4 vertices
- If different signs
- Line intersection



Naïve algorithm

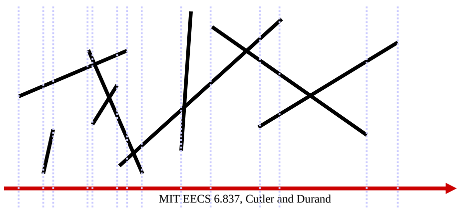
- N^2 intersection:

```
For (I=0; I<N; I++)
  For (J=I+1; J<N; J++)
    Compute intersection segments I and J
```



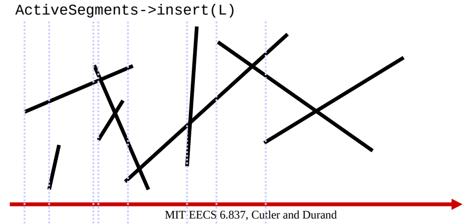
Taking advantage of coherence 1

- Sort in x
- Test only overlapping segments



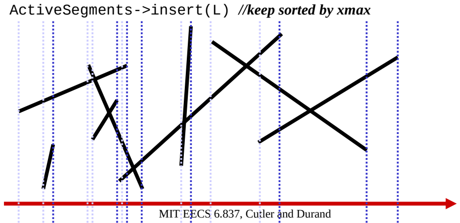
Taking advantage of coherence 1

```
Sort segments by xmin into queue Q
List ActiveSegments =empty
While Q not empty
   L= Q.next() //pick next segment
   ActiveSegment->removeSegmentsBefore(L.xmin)
   For all segments Li in Active segments
      Compute Intersection between L and Li
   ActiveSegments->insert(L)
```



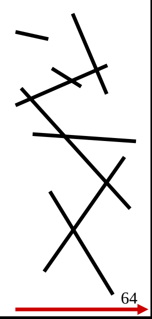
Taking advantage of coherence 1

```
Sort segments by xmin into queue Q
List ActiveSegments =empty
While Q not empty
   L= Q.next() //pick next segment
   ActiveSegment->removeSegmentsBefore(L.xmin) //easier if sorted
   For all segments Li in Active segments
      Compute Intersection between L and Li
   ActiveSegments->insert(L) //keep sorted by xmax
```



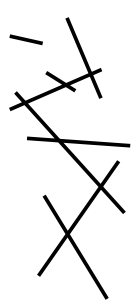
What have we achieved?

- Take advantage of locality and coherence
- Maintain working set
- Still O(n^2)
- But much better on average
- Can we do better?



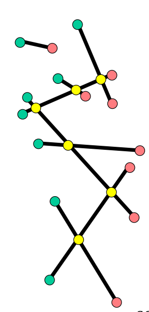
Can we do better?

- We have taken advantage of the coherence in x
- We have maintained a local view of the world at discrete events in x
- Do the same in y as well



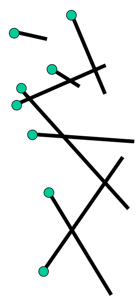
Maintain segments sorted in y

- Events
  - New segment
  - End of segment
  - Change of y sorting



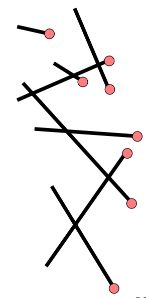
New segment

- Just insert at y1
- Use balanced binary trees



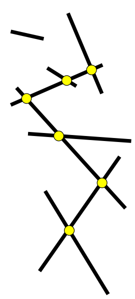
End of segment

- Just remove
- Potentially re-balance the tree



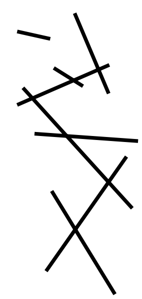
Intersection

- Where can intersection occur?
- Intersection must be between segments adjacents in y
- Fort each pair of adjacent segments, always maintain next intersection



Sweep algorithm

- Maintain event queue
  - New segment for each x1
    - Insert in binary tree
    - Compute potential new intersection
    - Add ending event
  - End of segment
    - simply remove
    - compute new intersections
  - Change of y sorting
    - report intersection
    - swap two segments
    - compute new intersections



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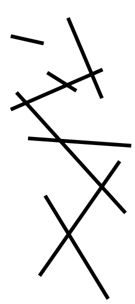

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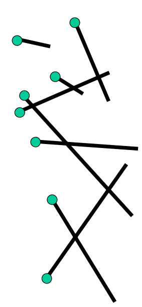

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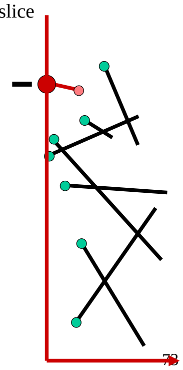

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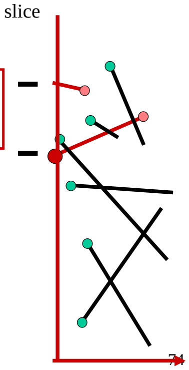

Sweep algorithm

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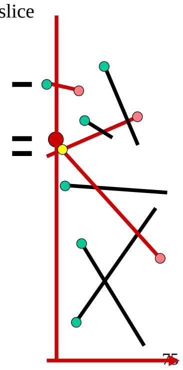

Sweep algorithm

- Maintain event queue
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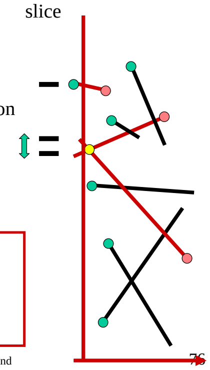
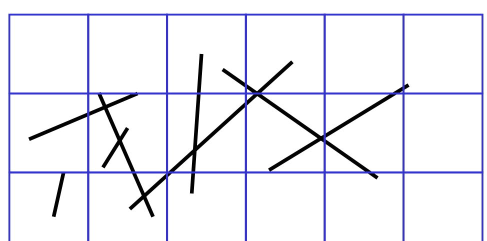


Output sensitive

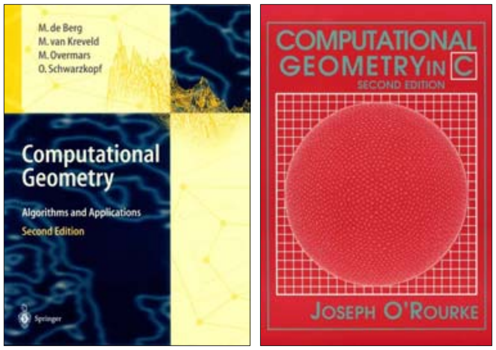
- The running time depends on the output
- Hopefully linear in the output
  + smaller complexity in the input
- In our case time O(n log n + k log n)
  - Where k is the number of intersections
- Space: O(n)
- The optimal bound is time O(n log n +k)

Other strategy?

- Grid!

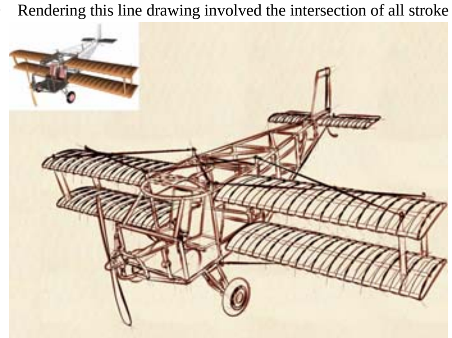


Ref



Questions?

- Rendering this line drawing involved the intersection of all stroke segments



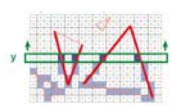
Plan

- Review of rendering pipeline

- 2D polygon clipping

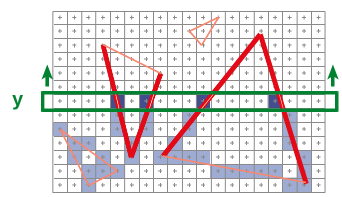
- Segment intersection

- Scanline rendering overview



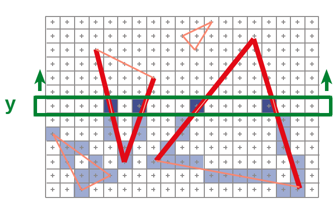
Scan Line rasterization

- Draw one scanline at a time
- Maintain ordered slices of triangles
- Advantage, does not require whole model and whole image in memory



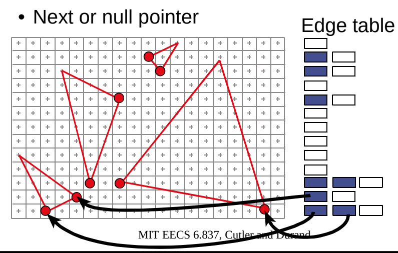
Scan Line : Principle

- Proceed row by row
- Maintain Active Edge List (AEL) (EdgeRecList)
- Edge Table (ET) for new edges at y (EdgeRecTable)



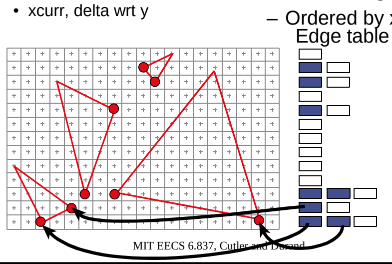
Precompute: Edge Table

- One entry per scan line (where edge begins)
- Each entry is a linked list of Edges, sorted by x
  - yend: y of top edge endpoint
  - xcurr, x: current x intersection, delta wrt y
  - Next or null pointer



Initialization: events

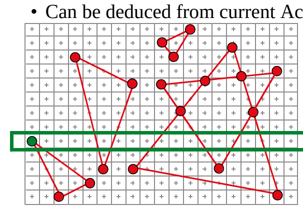
- Edge Table
  - List of Edges, sorted by x
    - yend
    - xcurr, delta wrt y
- Active edge list (AEL)
  - Will be maintained
  - Store all active edges intersecting scanline
  - Ordered by x



MIT EECS 6.837, Cutler and Durand 85

When Does AEL Change State?

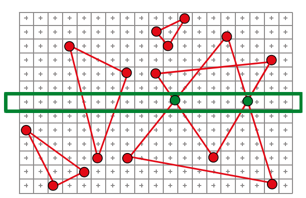
- When a vertex is encountered
  - When an edge begins
    - All such events pre-stored in Edge Table
  - When and edge ends
    - Can be deduced from current Active Edge List



MIT EECS 6.837, Cutler and Durand 86

When Does AEL Change State?

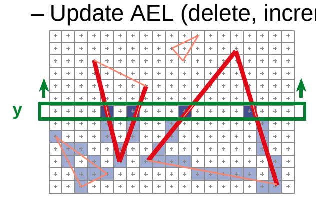
- When a vertex is encountered
- When two edges change order along a scanline
  - I.e., when edges cross each other!
  - How to detect this efficiently?



MIT EECS 6.837, Cutler and Durand 87

Scanline algorithm summary

- Initialize Raster, Polygons, Edge Table, AEL
- For each scanline y
  - Update Active Edge List (insert edges from EdgeTable[y])
  - Assign raster of pixels from AEL
  - Update AEL (delete, increment, resort)



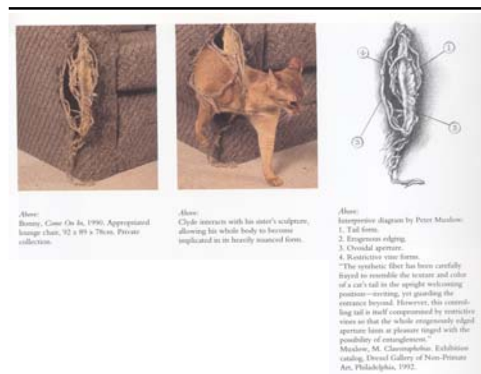
MIT EECS 6.837, Cutler and Durand 88

Other sweep algorithms

- Sweep is a very general principle:
  - Maintain a slice
  - Update at events
  - Works well if events are predictable locally in the slice (regular)
- Applied to many problems
  - E.g. construction of weird visibility data structures in 4.5D

MIT EECS 6.837, Cutler and Durand 89

Questions?



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